

Analysis and Control of Functional Brain Networks

Fabio Pasqualetti

October 16, 2023

Department of Mechanical Engineering
University of California at Riverside



Who really did the work



Tommaso Menara



Yuzhen Qin

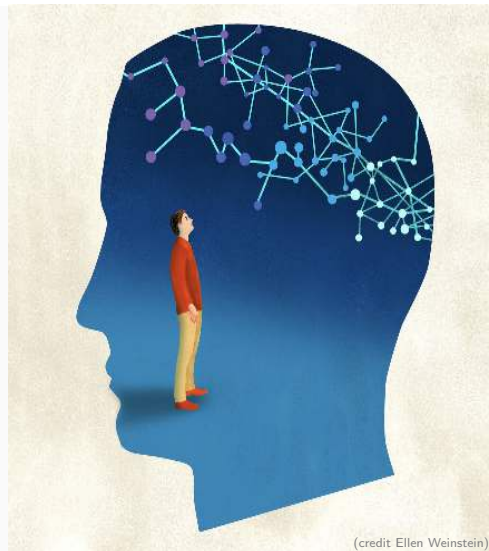


Giacomo Baggio

1/35

Reverse-Engineer the Human Brain

One of the greatest challenges of
modern science, and the key to ...



(credit Ellen Weinstein)

Why should we reverse-engineer the brain?

- personalized therapies & reversal of cognitive decline



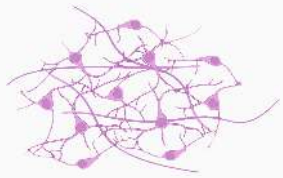
[Medtronic]

3/35

Why should we reverse-engineer the brain?

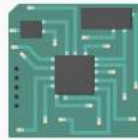
- personalized therapies & reversal of cognitive decline
- efficient computing systems & brain-computer interfaces

energy: ~ 10 W



vs

energy: ~ 100 W



[von Neumann, 2012, The Computer & the Brain]

3/35

Why should we reverse-engineer the brain?

- personalized therapies & reversal of cognitive decline
- efficient computing systems & brain-computer interfaces
- next generation machine intelligence



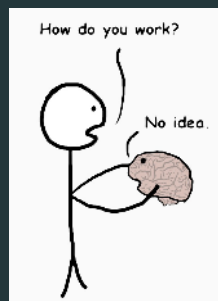
[Deepmind]



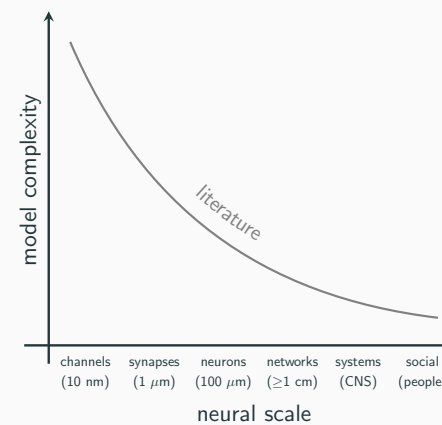
[Google Research]

3/35

How to reverse-engineer the brain?

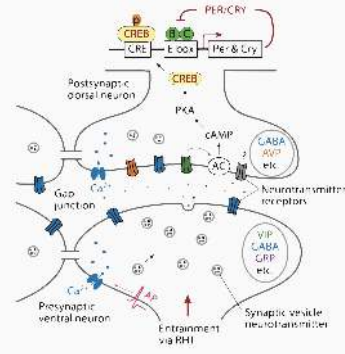
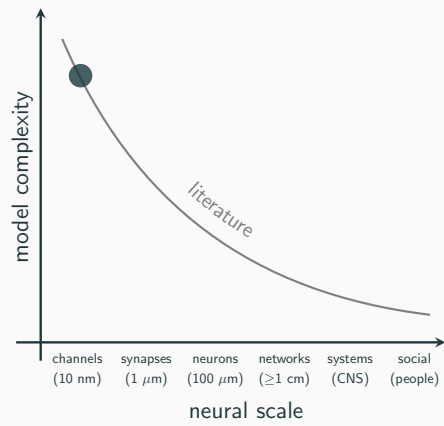


Models and scales of interest



4/35

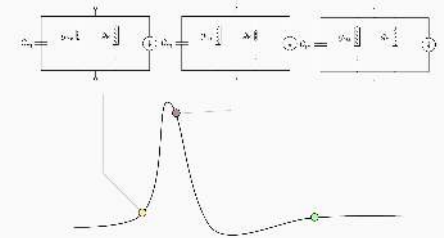
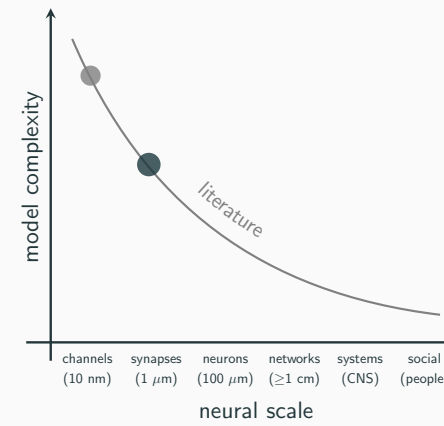
Models and scales of interest



[Welsh et al., 2010, Ann Rev Physiol]

4/35

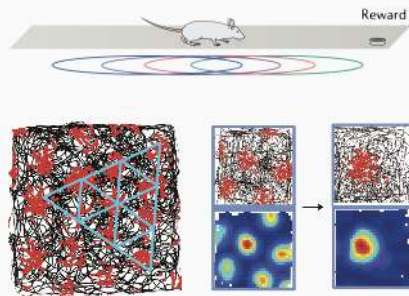
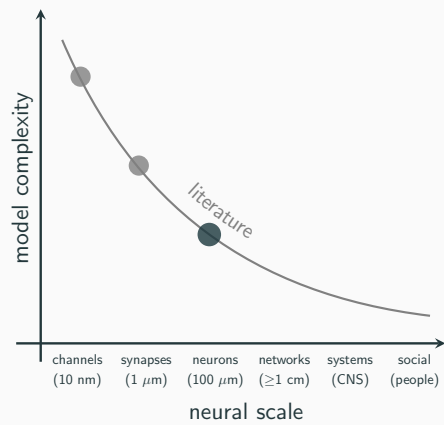
Models and scales of interest



[Kumar et al., 2010, Closed Loop Neuroscience]

4/35

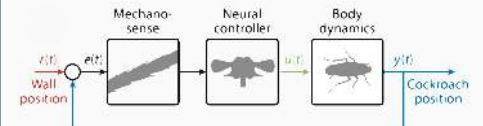
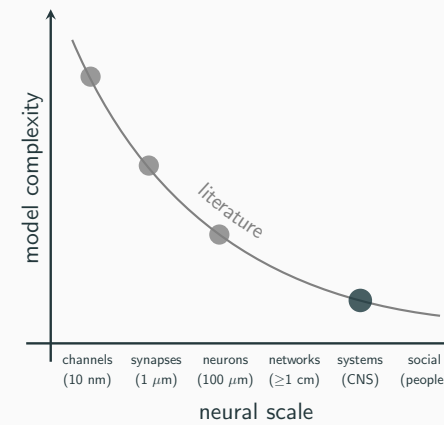
Models and scales of interest



[Moser et al., 2015, Cold Spring Harb Perspect Biol]

4/35

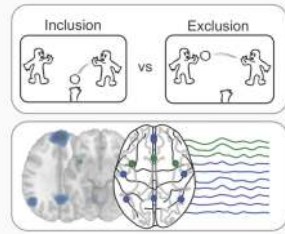
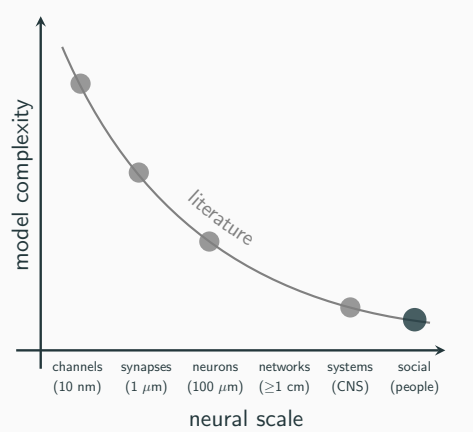
Models and scales of interest



[Madhav & Cowan, 2020 Ann Rev Contr Rob Aut Sys]

4/35

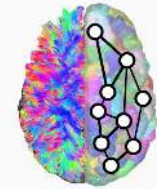
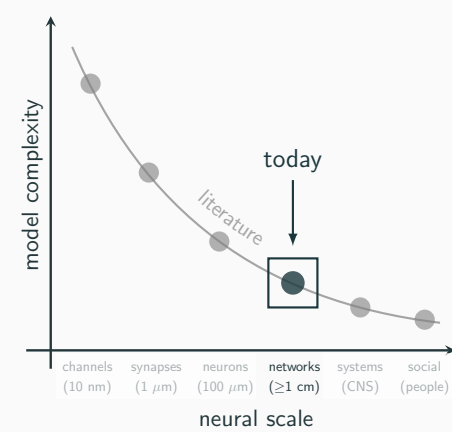
Models and scales of interest



[Schmälzle et al., 2017 PNAS]

4/35

Models and scales of interest



- brain-wide process (*functional patterns*)
- models retain biological compatibility
- models remain analytically tractable

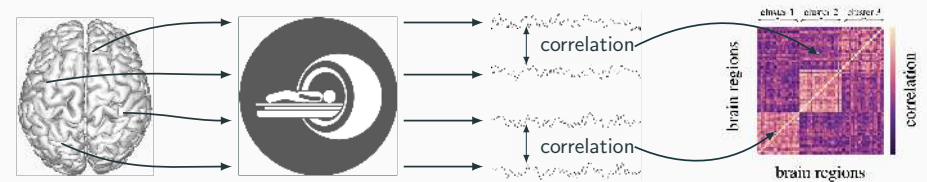
4/35

Functional patterns of brain activity



5/35

Functional patterns of brain activity

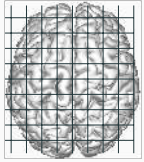


- synchrony between brain regions long known (EEG, fMRI, [Berger & Gray, 1929])
- rich repertoire of synchrony patters (transient, long-range, clustered)
- different patterns are biomarkers of health and disease (epilepsy, Parkinson's)

5/35

Modeling functional patterns

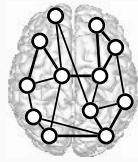
brain regions



white matter fibers



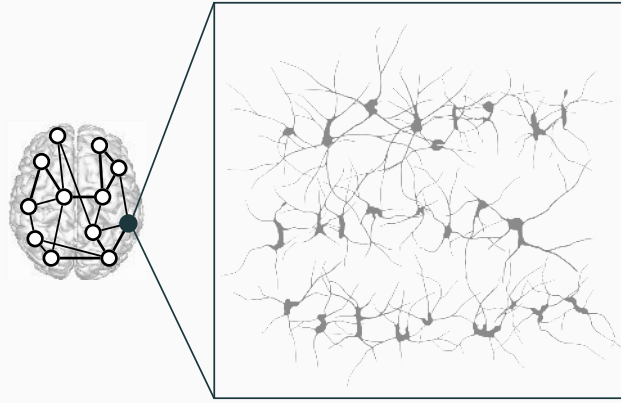
brain network



- nodes = brain regions; edges = bundles of white matter fibers
- static brain networks carry structural and statistical information
- *dynamic* brain networks are useful for the prediction & control of neural dynamics

6/35

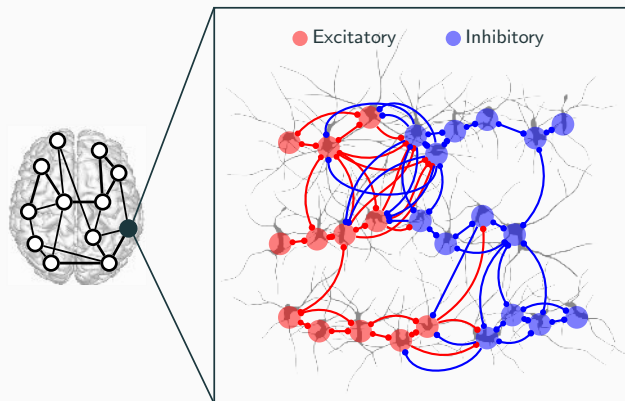
Resting-state activity modeled by phase oscillators



each node of the brain network captures the dynamics of a population of neurons

7/35

Resting-state activity modeled by phase oscillators



excitatory and inhibitory communities are in a regime of self-sustained oscillations
(weakly coupled Wilson-Cowan)
[Hoppensteadt and Izhikevich, 1997]

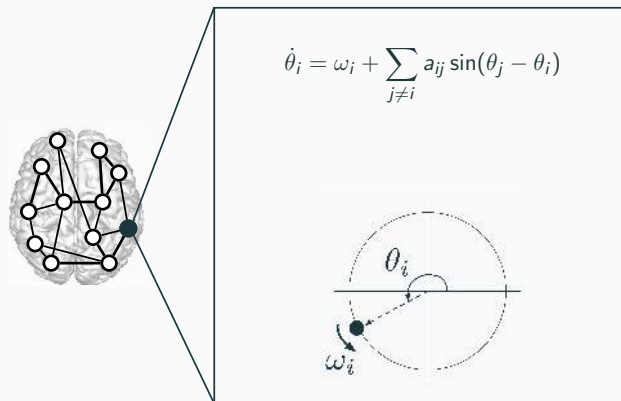
neurons' firing rates describe a limit cycle

dynamics approximated by a single phase variable

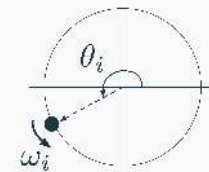
[Cabral et al., 2011]

7/35

Resting-state activity modeled by phase oscillators

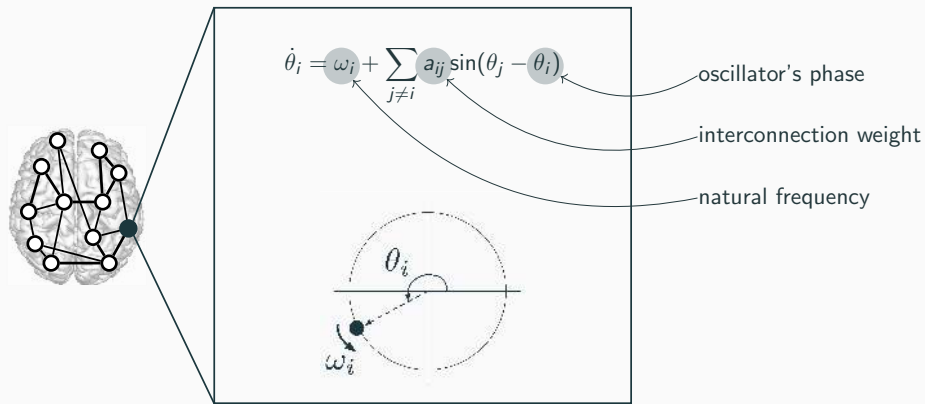


$$\dot{\theta}_i = \omega_i + \sum_{j \neq i} a_{ij} \sin(\theta_j - \theta_i)$$



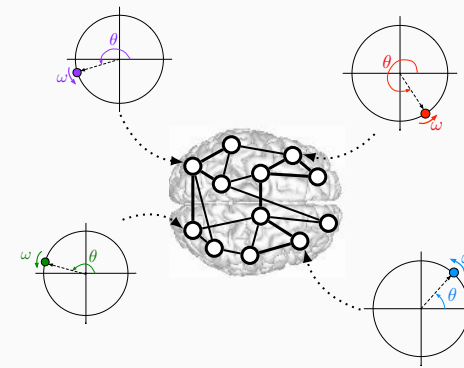
7/35

Resting-state activity modeled by phase oscillators



7/35

Dynamical brain network to simulate neural activity

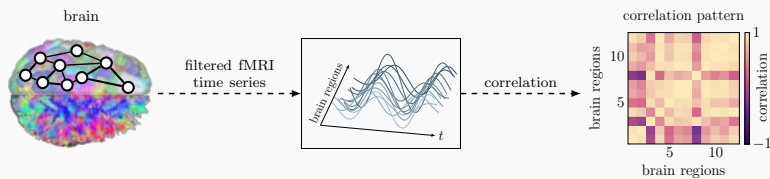


dynamical brain network with:

- nodes = brain regions
- edges = white matter fibers
- node dynamics = Kuramoto

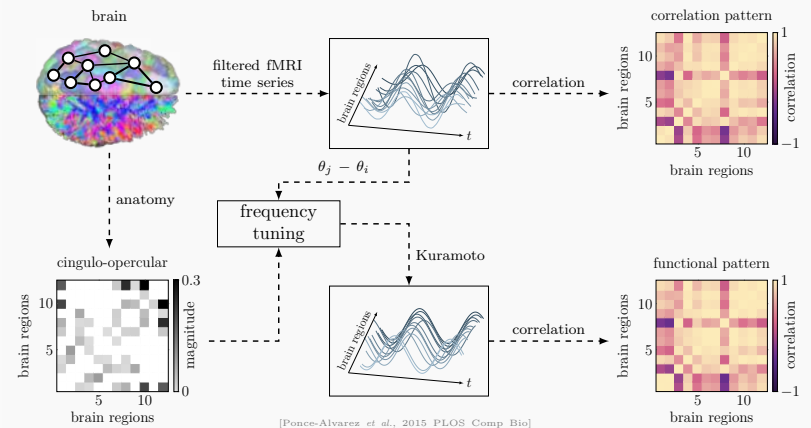
8/35

Sanity check



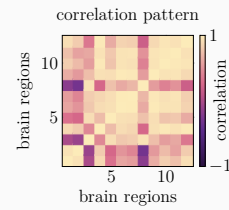
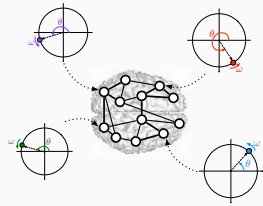
9/35

Sanity check



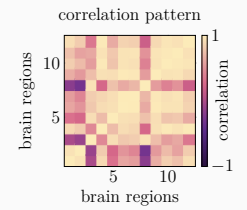
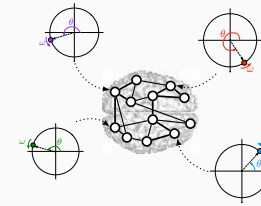
9/35

Oscillator properties and functional patterns



10/35

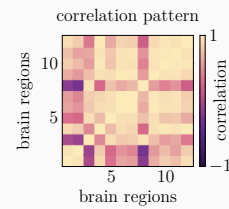
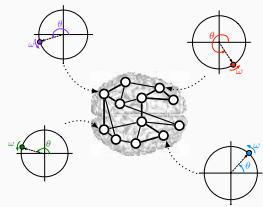
Oscillator properties and functional patterns



- correlated brain regions $\Leftrightarrow$ synchronized oscillators
- correlated regions typically form disjoint clusters

10/35

Oscillator properties and functional patterns



- correlated brain regions $\Leftrightarrow$ synchronized oscillators
- correlated regions typically form disjoint clusters

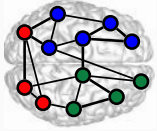
cluster synchronization in oscillator networks as a proxy for correlated neural activity

10/35

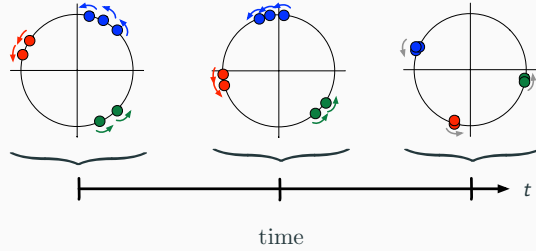
Analysis of cluster synchronization

Cluster synchronization

Network partition
 $\mathcal{P} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}$



$\Rightarrow$



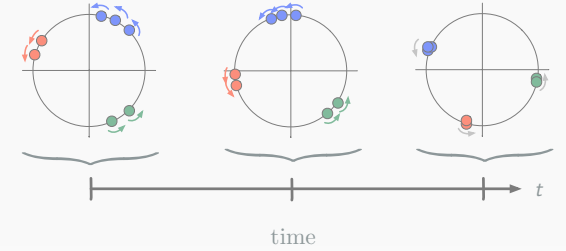
11/35

Cluster synchronization

Network partition
 $\mathcal{P} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}$



$\Rightarrow$

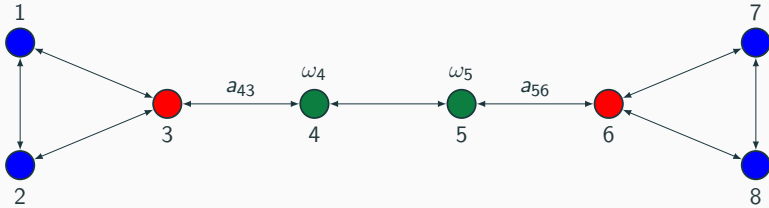


cluster synchronization manifold for $\mathcal{P} = \{\mathcal{C}_1, \dots, \mathcal{C}_m\}$:

$$\mathcal{S}_{\mathcal{P}} = \{\theta \in \mathbb{T}^n : \theta_i = \theta_j \text{ for all } i, j \in \mathcal{C}_k, k = 1, \dots, m\}$$

11/35

Invariance of cluster synchronization

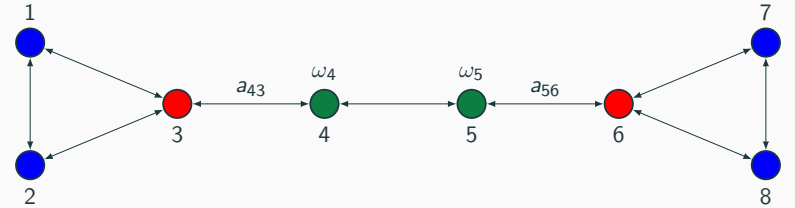


$$\dot{\theta}_4 = \omega_4 + a_{43} \sin(\theta_3 - \theta_4) + a_{45} \sin(\theta_5 - \theta_4)$$

$$\dot{\theta}_5 = \omega_5 + a_{56} \sin(\theta_6 - \theta_5) + a_{54} \sin(\theta_4 - \theta_5)$$

12/35

Invariance of cluster synchronization



$$\dot{\theta}_4 - \dot{\theta}_5 = \underbrace{\omega_4 - \omega_5}_{\text{diff. nat. freq.}} + \underbrace{a_{43} \sin(\theta_3 - \theta_4) - a_{56} \sin(\theta_6 - \theta_5)}_{\text{diff. external coupling}}$$

oscillators 4 and 5 remain phase synchronized if:

- diff. of natural frequencies = - diff. external coupling at all times
- equal natural frequencies ($\omega_4 = \omega_5$) and equal coupling ($a_{43} = a_{56}$)

12/35

Invariance of cluster synchronization

Invariance of the cluster synchronization manifold $\mathcal{S}_{\mathcal{P}}$ iff:

- $\omega_i = \omega_j$ for all oscillators in the same cluster
- the network weights are balanced

[Menara et al., 2020 TCNS]

balanced weights for partition $\mathcal{P} = \{\mathcal{C}_1, \mathcal{C}_2, \dots\}$:

$$\sum_{z \in \mathcal{C}_\ell} a_{iz} - a_{jz} = 0 \text{ for all } i, j \in \mathcal{C}_k \text{ and all partitions } \mathcal{C}_\ell \neq \mathcal{C}_k$$

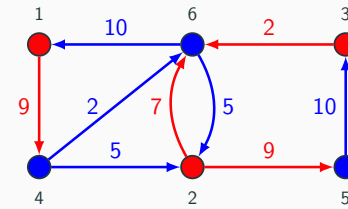
13/35

Invariance of cluster synchronization

Invariance of the cluster synchronization manifold $\mathcal{S}_{\mathcal{P}}$ iff:

- $\omega_i = \omega_j$ for all oscillators in the same cluster
- the network weights are balanced

[Menara et al., 2020 TCNS]



$$\mathcal{C}_1 = \{1, 2, 3\}, \quad \mathcal{C}_2 = \{4, 5, 6\}$$

$$\mathcal{C}_1 \rightarrow \mathcal{C}_2^i = \mathcal{C}_1 \rightarrow \mathcal{C}_2^j$$

$$\mathcal{C}_2 \rightarrow \mathcal{C}_1^i = \mathcal{C}_2 \rightarrow \mathcal{C}_1^j$$

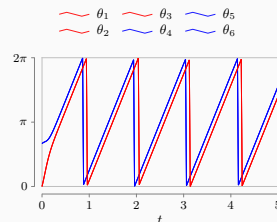
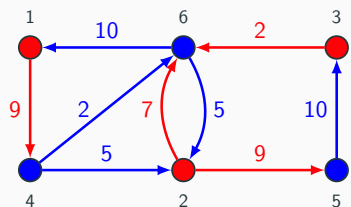
13/35

Invariance of cluster synchronization

Invariance of the cluster synchronization manifold $\mathcal{S}_{\mathcal{P}}$ iff:

- $\omega_i = \omega_j$ for all oscillators in the same cluster
- the network weights are balanced

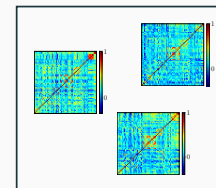
[Menara et al., 2020 TCNS]



13/35

Cluster invariance in empirical brain networks

fMRI data correlations



24 subjects
[Ponce-Alvarez et al.,
2015 PLOS Comp Bio]

clustering algorithm

Louvain, Ncut

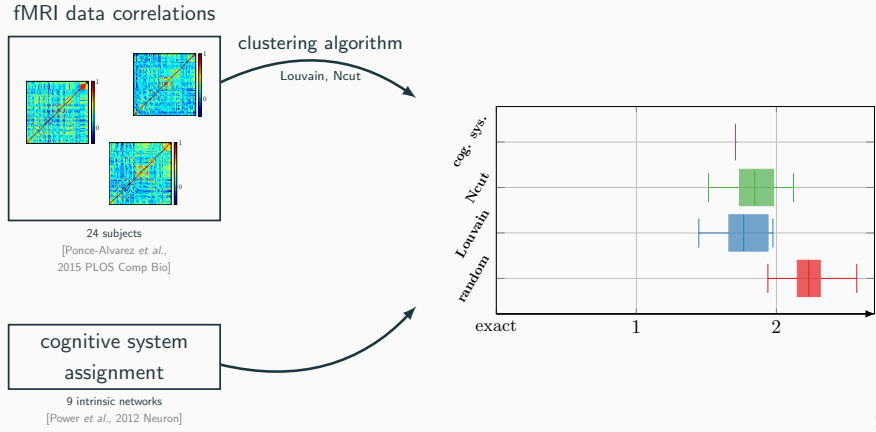
...distance from balanced network weights

cognitive system
assignment

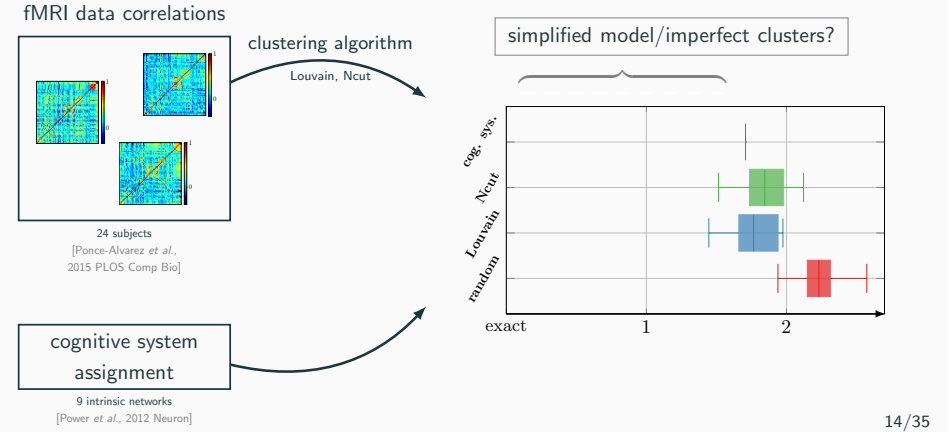
9 intrinsic networks
[Power et al., 2012 Neuron]

14/35

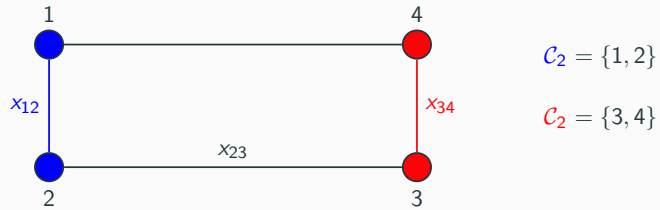
Cluster invariance in empirical brain networks



Cluster invariance in empirical brain networks



Local stability of cluster synchronization



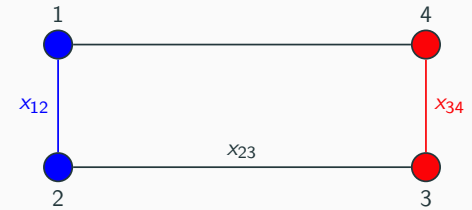
$$\dot{x}_{12} = \dot{\theta}_1 - \dot{\theta}_2 = -(a_{12} + a_{21}) \sin(x_{12}) + a_{13} \sin(x_{23}) - a_{13} \sin(x_{12} + x_{23} + x_{34})$$

$$\dot{x}_{34} = \dot{\theta}_3 - \dot{\theta}_4 = -(a_{34} + a_{43}) \sin(x_{34}) + a_{31} \sin(x_{23}) - a_{31} \sin(x_{12} + x_{23} + x_{34})$$

$$\dot{x}_{23} = \dot{\theta}_1 - \dot{\theta}_3 = (\omega_2 - \omega_1) + a_{43} \sin(x_{34}) + a_{21} \sin(x_{12}) - (a_{42} + a_{24}) \sin(x_{23})$$

15/35

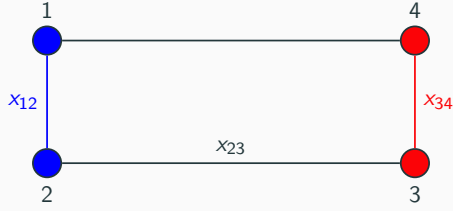
Local stability of cluster synchronization



$$\begin{pmatrix} \dot{x}_{12} \\ \dot{x}_{34} \end{pmatrix} = \underbrace{\begin{pmatrix} -(a_{12} + a_{21}) \sin(x_{12}) \\ -(a_{34} + a_{43}) \sin(x_{34}) \end{pmatrix}}_{F(x_{\text{intra}})} + \underbrace{(\sin(x_{23}) - \sin(x_{12} + x_{23} + x_{34}))}_{G(x_{\text{intra}}, x_{\text{inter}})} \begin{pmatrix} a_{13} \\ a_{31} \end{pmatrix}$$

15/35

Local stability of cluster synchronization

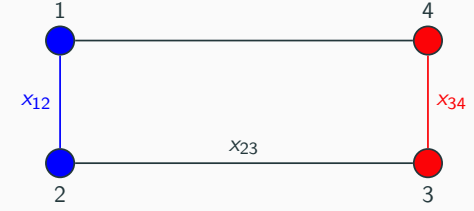


$$\dot{x}_{\text{intra}} = F(x_{\text{intra}}) + G(x_{\text{intra}}, x_{\text{inter}})$$

- F = intra-cluster dynamics; G = inter-cluster dynamics
- the origin of F is exponentially stable with rate dep. on intra-cluster weights
(from homogeneous Kuramoto dynamics)
- G is vanishing ($G(0, x_{\text{inter}}) = 0$) and lin. bounded ($\|G(x_{\text{intra}}, x_{\text{inter}})\| \leq \gamma \|x_{\text{intra}}\|$)

15/35

Local stability of cluster synchronization

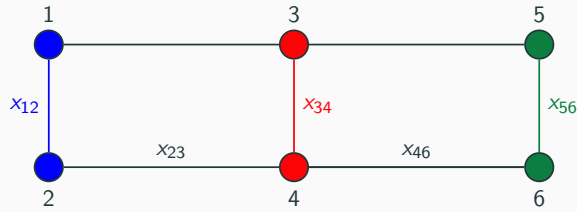


If intra-cluster weights $\gg$ inter-cluster weights, then $x_{\text{intra}} = 0$ is locally exponentially stable, and the cluster synchronization manifold is locally exponentially stable.

[Menara et al., 2020 TCNS]

15/35

Stability of multiple clusters



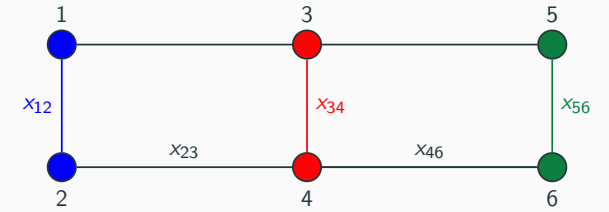
$$\dot{x}_{\text{intra}}^{(k)} = F^{(k)}(x_{\text{intra}}^{(k)}) + G^{(k)}(x_{\text{intra}}, x_{\text{inter}})$$

J_k : Jacobian of F at $x_{\text{intra}}^{(k)} = 0$ X_k : solution to $J_k^T X_k + X_k J_k = -I$

$$\gamma^{(k\ell)} : \|G^{(k)}(x_{\text{intra}}, x_{\text{inter}})\| \leq \sum_{\ell=1}^m \gamma^{(k\ell)} \|x_{\text{intra}}^{(\ell)}\|$$

16/35

Stability of multiple clusters



If the following matrix is an M -matrix,

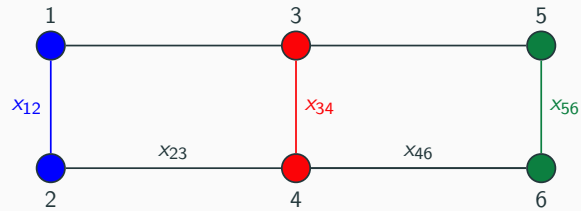
$$S = [s_{k\ell}] = \begin{cases} \lambda_{\max}^{-1}(X_k) - \gamma^{(kk)} & \text{if } k = \ell, \\ -\gamma^{(k\ell)} & \text{if } k \neq \ell, \end{cases}$$

then the synchronization manifold is locally exponentially stable.

[Menara et al., 2020 TCNS]

16/35

Stability of multiple clusters



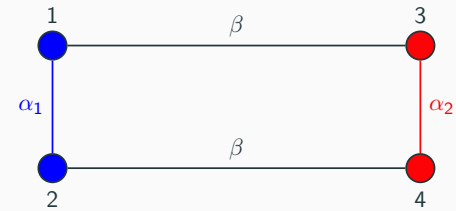
If the following matrix is an M -matrix,

intra-cluster weights $\gg$ inter-cluster weights $\Rightarrow$ stability

then the synchronization manifold is locally exponentially stable.

[Menara et al., 2020, TCNS] 16/35

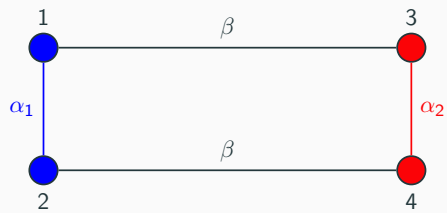
An example



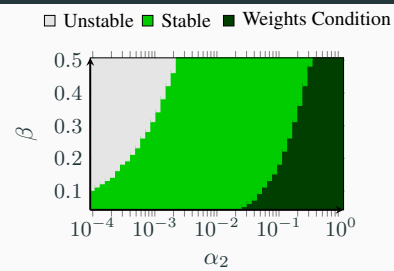
- fix $\alpha_1, \omega_1, \omega_2$ and vary α_2, β

17/35

An example

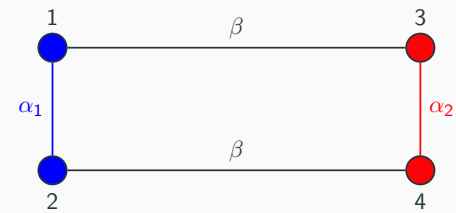


- fix $\alpha_1, \omega_1, \omega_2$ and vary α_2, β

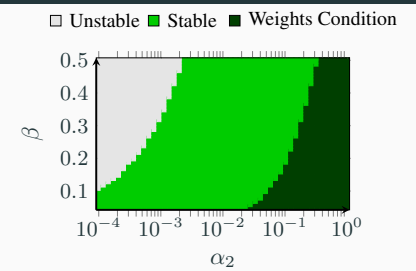


17/35

An example

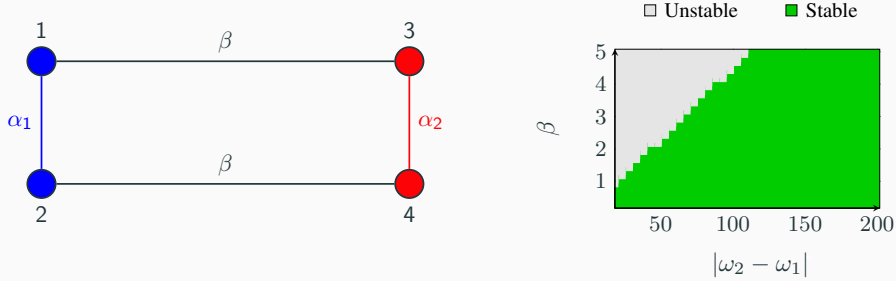


- fix $\alpha_1, \omega_1, \omega_2$ and vary α_2, β
- network weights provide conservative estimates of stability. frequencies?



17/35

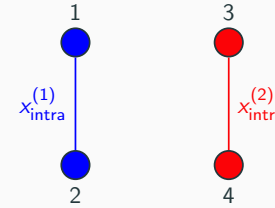
An example



- fix α_1, α_2 , and vary $\beta, |\omega_2 - \omega_1|$
- network weights provide conservative estimates of stability. frequencies?
- large frequency differences promote stability. why?

17/35

Approximate stability of cluster synchronization



- isolated clusters are stable oscillatory systems

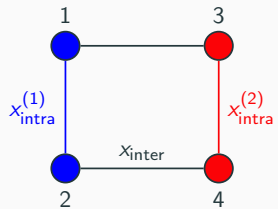
- linearized dynamics:

$$\dot{x}_{\text{intra}}^{(1)} = J_1 x_{\text{intra}}^{(1)}$$

- connectivity + homogeneous frequencies $\Rightarrow$ intra-cluster synchronization

18/35

Approximate stability of cluster synchronization



- isolated clusters are stable oscillatory systems
- clusters subject to inter-cluster perturbation

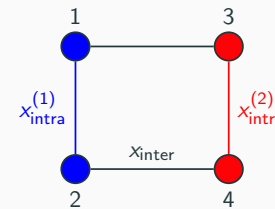
- linearized dynamics:

$$\dot{x}_{\text{intra}}^{(1)} = J_1 x_{\text{intra}}^{(1)} + \eta_{12} \cos(x_{\text{inter}}) x_{\text{intra}}^{(2)}$$

- $x_{\text{inter}} \rightarrow (\omega_2 - \omega_1)t$ as $|\omega_2 - \omega_1|$ grows

18/35

Approximate stability of cluster synchronization



- isolated clusters are stable oscillatory systems
- clusters subject to inter-cluster perturbation

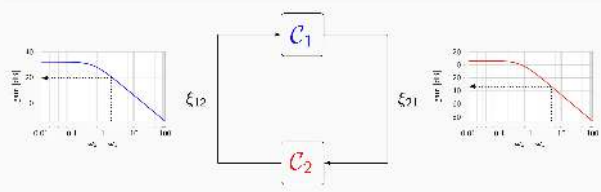
- linearized dynamics:

$$\dot{x}_{\text{intra}}^{(1)} \approx J_1 x_{\text{intra}}^{(1)} + \eta_{12} \cos((\omega_2 - \omega_1)t) x_{\text{intra}}^{(2)}$$

- inter-cluster perturbation is modulated by $\omega_2 - \omega_1$

18/35

Approximate stability of cluster synchronization



18/35

Approximate stability of cluster synchronization

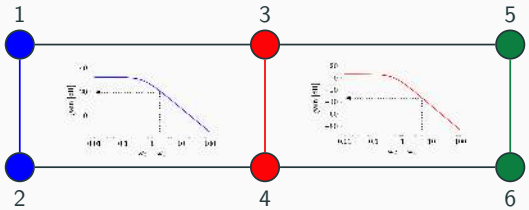


If $|\omega_j - \omega_i| \rightarrow \infty$ for all clusters, then the synchronization manifold is locally exponentially stable.

[Menara et al., 2020 TCNS]

18/35

Stability of multiple clusters using weights and frequencies



$\xi_{ij} = | \text{frequency gain from cluster } C_i \text{ to cluster } C_j |$

If $\lambda_{\max}([\xi_{ij}]) < 1$, then the sync. manifold is locally exp. stable.

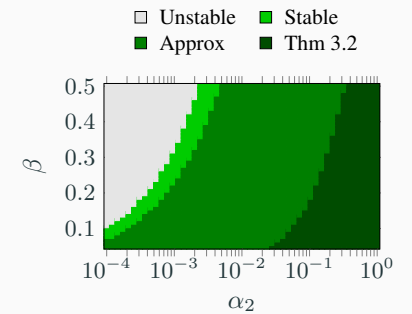
[Menara et al., 2019 ACC]

19/35

An example



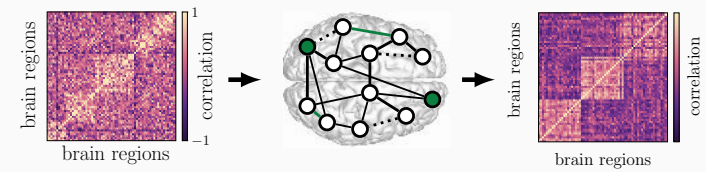
- fix $\alpha_1, \omega_1, \omega_2$ and vary α_2, β
- combine weights and frequency gains into small-gain condition
- combined weights/frequency conditions are consistently tight



20/35

Control of cluster synchronization

Control of functional patterns



so far...

- modeling of neural activity through oscillator network
- modeling of functional patterns via cluster synchronization
- conditions for invariance/stability of cluster synchronization

21/35

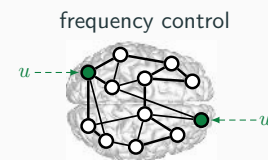
Control of functional patterns



how do we restore healthy functional patterns?

21/35

Control of functional patterns

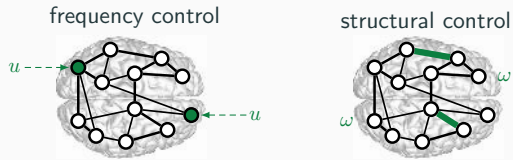


- external control of oscillator frequency

[Menara et al., 2020 LCSS]

22/35

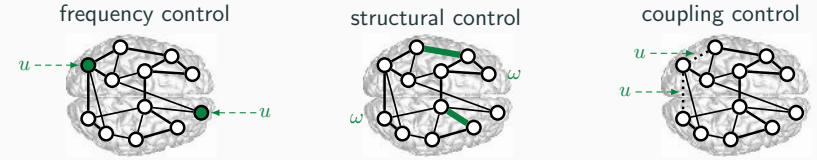
Control of functional patterns



- external control of oscillator frequency [Menara et al., 2020 LCSS]
- design of structural weights and oscillator frequencies [Menara et al., 2019 CDC & 2022 NatComm]

22/35

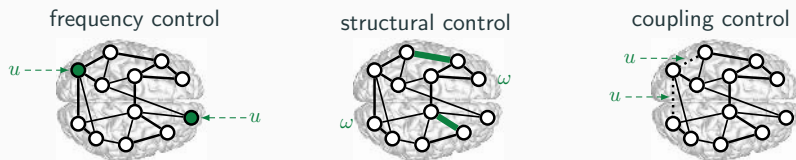
Control of functional patterns



- external control of oscillator frequency [Menara et al., 2020 LCSS]
- design of structural weights and oscillator frequencies [Menara et al., 2019 CDC & 2022 NatComm]
- external control of oscillators coupling [Qin et al., 2022 CDC]

22/35

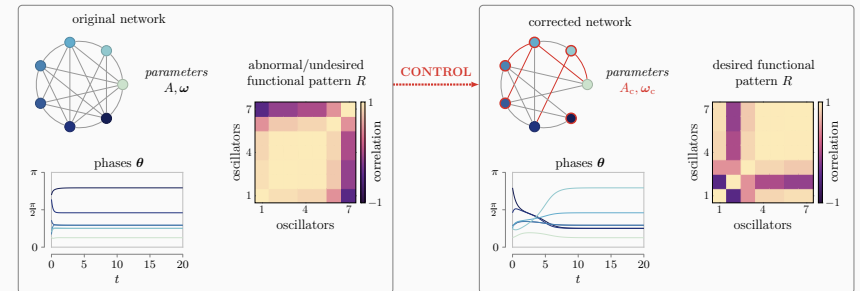
Control of functional patterns



- external control of oscillator frequency [Menara et al., 2020 LCSS]
- design of structural weights and oscillator frequencies [Menara et al., 2019 CDC & 2022 NatComm]
- external control of oscillators coupling [Qin et al., 2022 CDC]

22/35

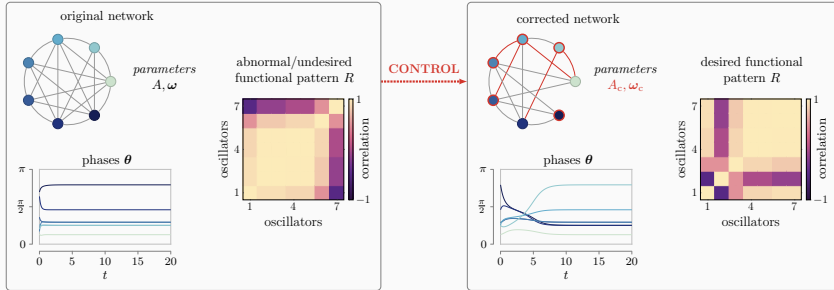
Structural control of functional patterns



- control knobs = network weights + oscillator frequencies
- biological constraints: positive weights, sparsity of interventions
- reference signal is $n \times n$ matrix of the phase correlation values (time-varying)

23/35

Structural control of functional patterns



- control knobs = network weights + oscillator frequencies
 - biological constraints: positive weights, sparsity of interventions
 - reference signal is $n \times n$ matrix of the phase correlation values (time-varying)
- focus on time-invariant patterns, equilibrium assignment ^{23/35}

Frequency-synchronization and functional patterns

network dynamics in matrix form (B = incidence matrix):

$$\begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_n \end{bmatrix} - B \begin{bmatrix} \ddots & & \\ & \sin(\theta_j - \theta_i) & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ a_{ij} \\ \vdots \end{bmatrix}$$

24/35

Frequency-synchronization and functional patterns

network dynamics in matrix form (B = incidence matrix):

$$\begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_n \end{bmatrix} - B \begin{bmatrix} \ddots & & \\ & \sin(\theta_j - \theta_i) & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ a_{ij} \\ \vdots \end{bmatrix}$$

when oscillators are frequency-synchronized:

- oscillator frequencies are all equal to $\omega_{\text{mean}} = \frac{1}{n} \sum \omega_i$
- functional correlations are defined by phase differences
- feasible functional patterns have only $n - 1$ degrees of freedom

24/35

Frequency-synchronization and functional patterns

frequency-synchronized configuration:

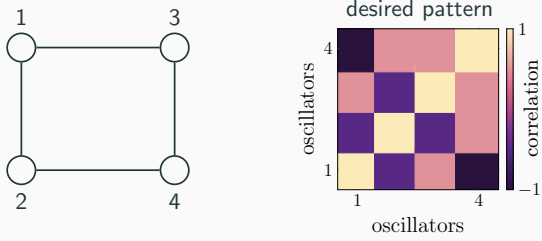
$$B \begin{bmatrix} \ddots & & \\ & \sin(\theta_j - \theta_i) & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ a_{ij} \\ \vdots \end{bmatrix} = \begin{bmatrix} \omega_1 - \omega_{\text{mean}} \\ \vdots \\ \omega_n - \omega_{\text{mean}} \end{bmatrix}$$

to generate a desired functional pattern:

- compute $n - 1$ phase differences corresponding to desired functional values
- determine feasibility of the desired equilibrium (sign/sparsity constraints)
- find network weights and frequencies to satisfy the above equation

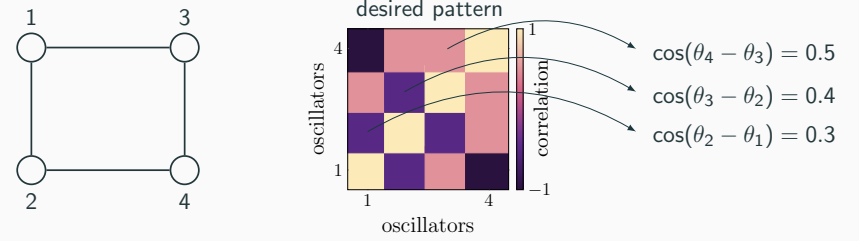
24/35

Feasibility of a functional pattern



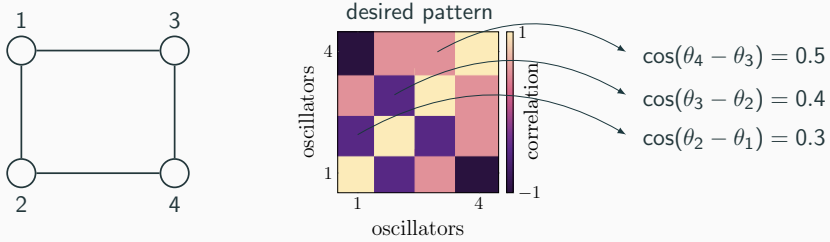
25/35

Feasibility of a functional pattern



25/35

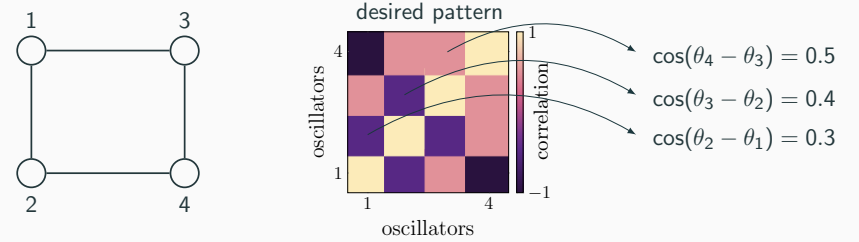
Feasibility of a functional pattern



$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sin(\theta_2 - \theta_1) \\ \sin(\theta_3 - \theta_2) \\ \sin(\theta_4 - \theta_3) \\ \sin(\theta_1 - \theta_4) \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{23} \\ a_{34} \\ a_{41} \end{bmatrix} = \begin{bmatrix} \omega_1 - \omega_{\text{mean}} \\ \omega_2 - \omega_{\text{mean}} \\ \omega_3 - \omega_{\text{mean}} \\ \omega_4 - \omega_{\text{mean}} \end{bmatrix}$$

25/35

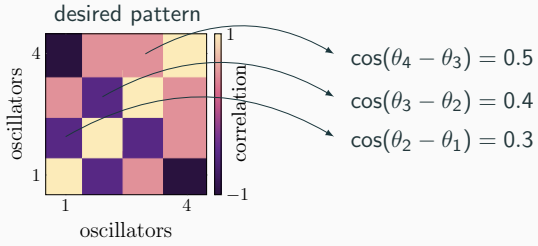
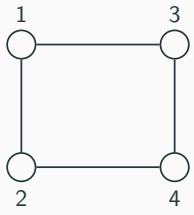
Feasibility of a functional pattern



$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sin(\theta_2 - \theta_1) \\ \sin(\theta_3 - \theta_2) \\ \sin(\theta_4 - \theta_3) \\ \sin(\theta_1 - \theta_4) \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{23} \\ a_{34} \\ a_{41} \end{bmatrix} = \begin{bmatrix} \omega_1 - \omega_{\text{mean}} \\ \omega_2 - \omega_{\text{mean}} \\ \omega_3 - \omega_{\text{mean}} \\ \omega_4 - \omega_{\text{mean}} \end{bmatrix} \quad \text{given}$$

25/35

Feasibility of a functional pattern



$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sin(\theta_2 - \theta_1) \\ \sin(\theta_3 - \theta_2) \\ \sin(\theta_4 - \theta_3) \\ \sin(\theta_1 - \theta_4) \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{23} \\ a_{34} \\ a_{41} \end{bmatrix} = \begin{bmatrix} \omega_1 - \omega_{\text{mean}} \\ \omega_2 - \omega_{\text{mean}} \\ \omega_3 - \omega_{\text{mean}} \\ \omega_4 - \omega_{\text{mean}} \end{bmatrix}$$

25/35

Feasibility of a functional pattern with positive weights

$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sin(\theta_2 - \theta_1) \\ \sin(\theta_3 - \theta_2) \\ \sin(\theta_4 - \theta_3) \\ \sin(\theta_1 - \theta_4) \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{23} \\ a_{34} \\ a_{41} \end{bmatrix} = \begin{bmatrix} \omega_1 - \omega_{\text{mean}} \\ \omega_2 - \omega_{\text{mean}} \\ \omega_3 - \omega_{\text{mean}} \\ \omega_4 - \omega_{\text{mean}} \end{bmatrix}$$

26/35

Feasibility of a functional pattern with positive weights

scaled incidence matrix $\bar{B}$

$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sin(\theta_2 - \theta_1) \\ \sin(\theta_3 - \theta_2) \\ \sin(\theta_4 - \theta_3) \\ \sin(\theta_1 - \theta_4) \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{23} \\ a_{34} \\ a_{41} \end{bmatrix} = \begin{bmatrix} \omega_1 - \omega_{\text{mean}} \\ \omega_2 - \omega_{\text{mean}} \\ \omega_3 - \omega_{\text{mean}} \\ \omega_4 - \omega_{\text{mean}} \end{bmatrix}$$

26/35

Feasibility of a functional pattern with positive weights

scaled incidence matrix $\bar{B}$

$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sin(\theta_2 - \theta_1) \\ \sin(\theta_3 - \theta_2) \\ \sin(\theta_4 - \theta_3) \\ \sin(\theta_1 - \theta_4) \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{23} \\ a_{34} \\ a_{41} \end{bmatrix} = \begin{bmatrix} \omega_1 - \omega_{\text{mean}} \\ \omega_2 - \omega_{\text{mean}} \\ \omega_3 - \omega_{\text{mean}} \\ \omega_4 - \omega_{\text{mean}} \end{bmatrix}$$

The functional pattern is feasible with pos. weights if:

- the network $\bar{B}$ contains a Hamiltonian path $\mathcal{H}$
- $\omega^\top \bar{B}_{\mathcal{H}} > 0$

[Menara et al., 2022 NatComm]

26/35

Feasibility of a functional pattern with positive weights

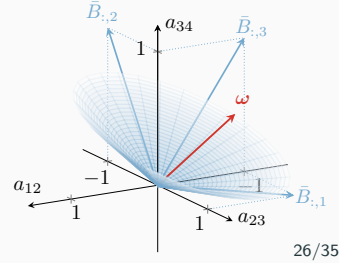
scaled incidence matrix $\bar{B}$

$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sin(\theta_2 - \theta_1) \\ \sin(\theta_3 - \theta_2) \\ \sin(\theta_4 - \theta_3) \\ \sin(\theta_1 - \theta_4) \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{23} \\ a_{34} \\ a_{41} \end{bmatrix} = \begin{bmatrix} \omega_1 - \omega_{\text{mean}} \\ \omega_2 - \omega_{\text{mean}} \\ \omega_3 - \omega_{\text{mean}} \\ \omega_4 - \omega_{\text{mean}} \end{bmatrix}$$

The functional pattern is feasible with pos. weights if:

- the network $\bar{B}$ contains a Hamiltonian path $\mathcal{H}$
- $\omega^\top \bar{B}_{\mathcal{H}} > 0$

[Menara et al., 2022 NatComm]



26/35

Feasibility of a functional pattern with positive weights

scaled incidence matrix $\bar{B}$

$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sin(\theta_2 - \theta_1) \\ \sin(\theta_3 - \theta_2) \\ \sin(\theta_4 - \theta_3) \\ \sin(\theta_1 - \theta_4) \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{23} \\ a_{34} \\ a_{41} \end{bmatrix} = \begin{bmatrix} \omega_1 - \omega_{\text{mean}} \\ \omega_2 - \omega_{\text{mean}} \\ \omega_3 - \omega_{\text{mean}} \\ \omega_4 - \omega_{\text{mean}} \end{bmatrix}$$

The functional pattern is feasible with pos. weights if:

- the network $\bar{B}$ contains a Hamiltonian path $\mathcal{H}$
- $\omega^\top \bar{B}_{\mathcal{H}} > 0$

[Menara et al., 2022 NatComm]

If pattern is feasible, solve (convex):

$$\begin{aligned} &\min J(a) \\ &\text{s.t. } a > 0 \text{ and } \bar{B}a = \omega \end{aligned}$$

26/35

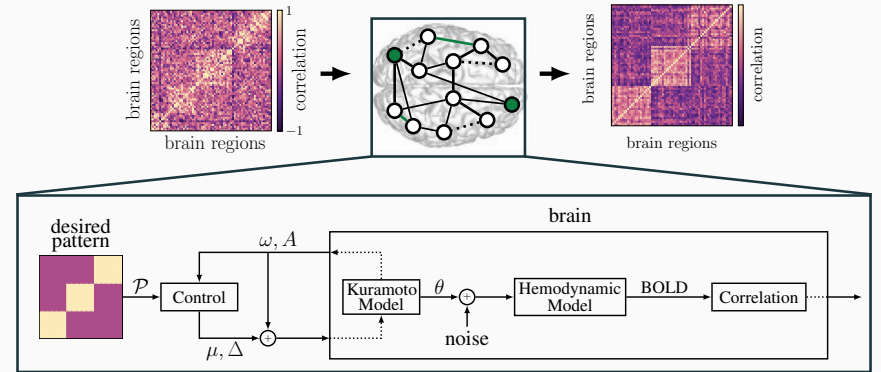
Restoring functional connectivity in the damaged brain



- structural data from Human Connectome Project
- (synthetic) functional data inspired by brain injury
- Balloon-Windkessel hemodynamic model for BOLD signals

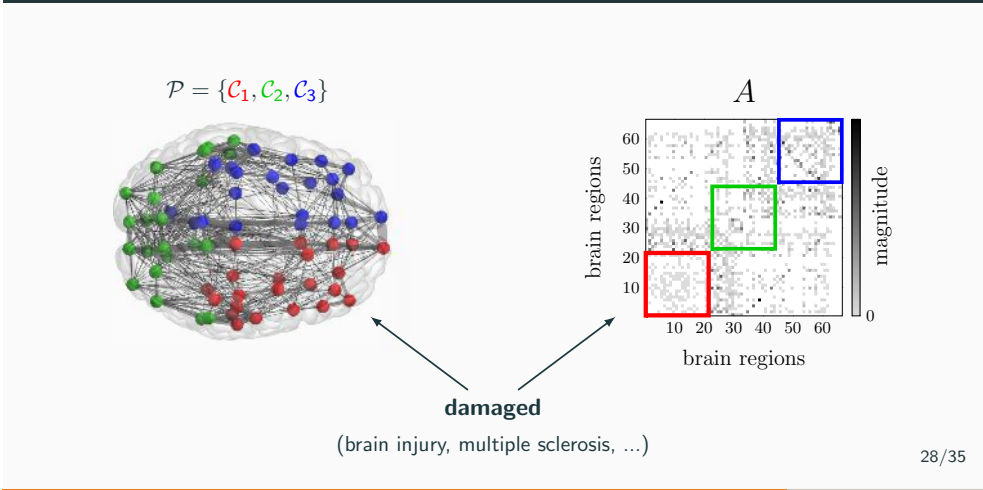
27/35

Restoring functional connectivity in the damaged brain

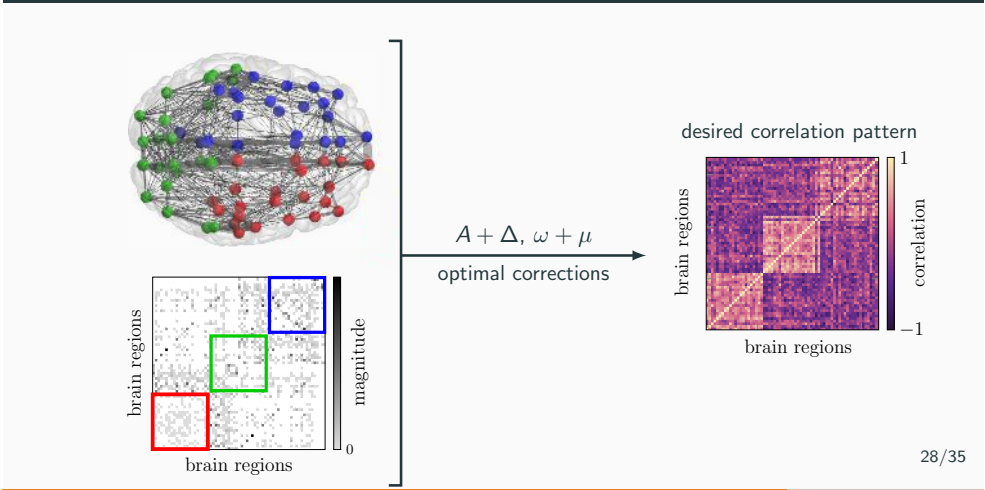


27/35

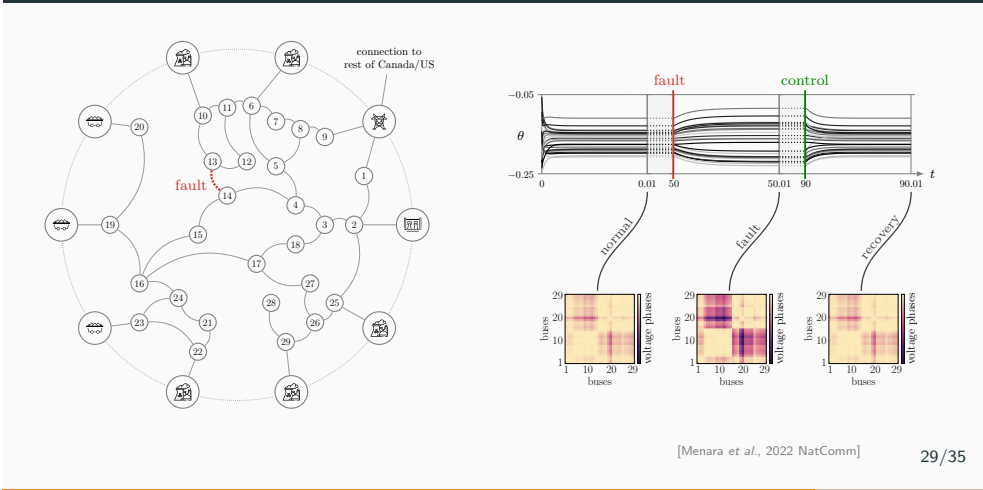
Restoring functional connectivity in the damaged brain



Restoring functional connectivity in the damaged brain



Beyond brain networks: power redistribution and fault recovery



Summary

Modeling, analysis, control of functional connectivity via cluster synchronization:

Summary

Modeling, analysis, control of functional connectivity via cluster synchronization:

Invariance: **balanced weights + homogeneous intra-cluster frequencies**

30/35

Summary

Modeling, analysis, control of functional connectivity via cluster synchronization:

Invariance: **balanced weights + homogeneous intra-cluster frequencies**

Stability: **intra-cluster coupling $\gg$ inter-cluster coupling**
large inter-cluster frequency differences
weights + frequencies $\Rightarrow$ tight small-gain conditions

30/35

Summary

Modeling, analysis, control of functional connectivity via cluster synchronization:

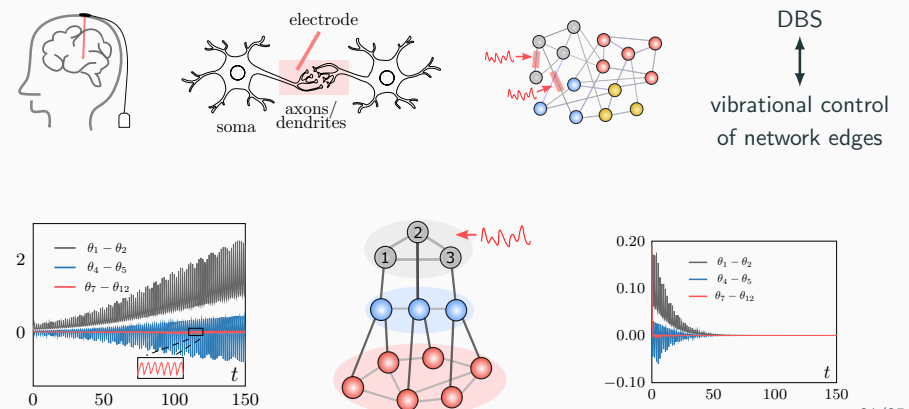
Invariance: **balanced weights + homogeneous intra-cluster frequencies**

Stability: **intra-cluster coupling $\gg$ inter-cluster coupling**
large inter-cluster frequency differences
weights + frequencies $\Rightarrow$ tight small-gain conditions

Control: **graph-theoretic cond. for feasibility of functional patterns**
structural control of functional patterns in brain/power

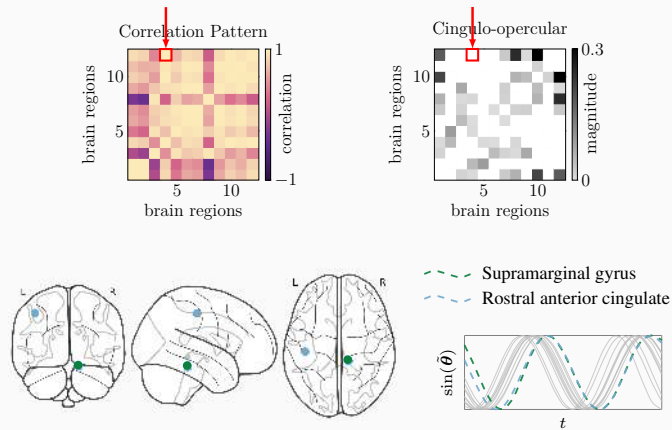
30/35

External control of oscillators coupling



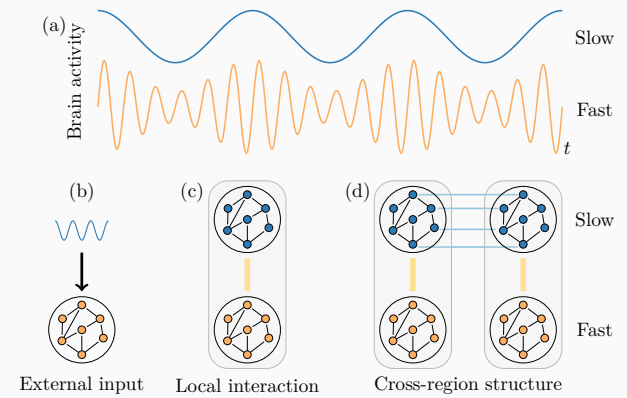
31/35

Remote (long-range) synchronization



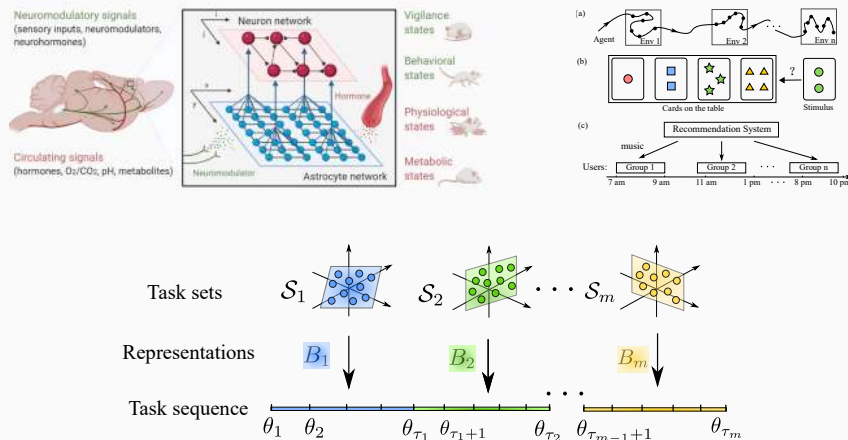
32/35

Phase-amplitude synchronization



33/35

Brain-inspired, context-aware reinforcement learning



34/35

References and acknowledgements



35/35

Analysis and Control of Functional Brain Networks

Fabio Pasqualetti

October 16, 2023

Department of Mechanical Engineering
University of California at Riverside

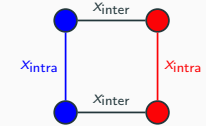
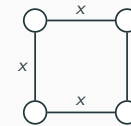


Full vs cluster synchronization

Phase differences: $x_{ij} = \theta_j - \theta_i$

full synchronization: $x \rightarrow 0$

cluster synchronization: $x_{\text{intra}} \rightarrow 0$
 $x_{\text{inter}} = ?$



35/35

Full vs cluster synchronization

Phase differences: $x_{ij} = \theta_j - \theta_i$

full synchronization: $x \rightarrow 0$

cluster synchronization: $x_{\text{intra}} \rightarrow 0$
 $x_{\text{inter}} = ?$

◇ Difference dynamics
 $\dot{x} = F(x)$

◇ Difference dynamics
 $\dot{x}_{\text{intra}} = F(x_{\text{intra}}) + G(x_{\text{intra}}, x_{\text{inter}})$

35/35

Full vs cluster synchronization

Phase differences: $x_{ij} = \theta_j - \theta_i$

full synchronization: $x \rightarrow 0$

cluster synchronization: $x_{\text{intra}} \rightarrow 0$
 $x_{\text{inter}} = ?$

◇ Difference dynamics
 $\dot{x} = F(x)$

◇ Linearization around synchronized trajectory
 $\dot{x} = Jx$

known ☺

unknown ☹

◇ Difference dynamics
 $\dot{x}_{\text{intra}} = F(x_{\text{intra}}) + G(x_{\text{intra}}, x_{\text{inter}})$

◇ Linearization around cluster-synchronized trajectory
 $\dot{x}_{\text{intra}} = (J_{\text{intra}} + J_{\text{inter}}(t))x_{\text{intra}}$

Hurwitz ☺

time-varying ☹ 35/35